DOCUMENT RESUME

ED 021 726 SE 004 505

By- Foley, Jack L.

NUMBER SENTENCES, EQUATIONS AND INEQUALITIES

Pub Date Aug 67

Note- 35p.

EDRS Price MF-$0.25 HC-$1.48

Descriptors- *ALGEBRA, *ARITHMETIC, CURRICULUM, *CURRICULUM DEVELOPMENT, ELEMENTARY SCHOOL MATHEMATICS, INSTRUCTIONAL MATERIALS, *MATHEMATICS, *SECONDARY SCHOOL MATHEMATICS

Identifiers- Elementary and Secondary Education Act Title III

This booklet, one of a series, has been developed for the project, A Program for Mathematically Underdeveloped Pupils. A project team, including inservice teachers, is being used to write and develop the materials for this program. The materials developed in this booklet include (1) number expressions, (2) symbols and ideas, (3) open equations, (4) combining "like" terms, (5) writing equivalent equations, (6) inequalities, and (7) illustration of terms. (RP)

NUMBER SENTENCES

— EQUATIONS AND INEQUALITIES —

ESEA Title III

PROJECT MATHEMATICS

Project Team

Dr. Jack L. Foley, Director
Elizabeth Basten, Administrative Assistant
Ruth Bower, Assistant Coordinator
Wayne Jacobs, Assistant Coordinator
Gerald Burke, Assistant Coordinator
Leroy B. Smith, Mathematics Coordinator for Palm Beach County

Graduate and Student Assistants

Jean Cruise
Kathleen Whittier
Jeanne Hullihan
Barbara Miller
Larry Hood

Donnie Anderson
Connie Speaker
Ambie Vought
Dale McClung

Secretaries

Novis Kay Smith
Dianah Hills
Juanita Wyne

TEACHERS

Sister Margaret Arthur
Mr. John Atkins, Jr.
Mr. Lawrence Bernier
Mr. Harry Berryman
Mr. Ricke Brown
Mrs. Nicola Corbin
Mrs. Gertrude Dixon
Mrs. Dellah Evans
Mrs. Marilyn Floyd
Mrs. Katherine Graves
Mrs. Aleen Harris
Mr. Earl I. Hawk
Mr. Arthur Herd
Mrs. Alice Houlihan
Mr. Harold Kerttula
Mrs. Mary Kisko
Mrs. Christine Maynor
Mr. Ladell Morgan
Mr. Charles G. Owen
Mrs. Margaret Patterson
Sister Ann Richard
Mr. Carl Sandifer
Mrs. Elizabeth Staley
Mr. James Stone
Mrs. Linda G. Teer
Mr. James Wadlington
Mrs. Marie Wells
Mr. Ronald Whitehead
Mrs. Mattie Whitfield
Mr. James Williams
Mr. Kelly Williams
Mr. Lloyd Williams

August, 1967

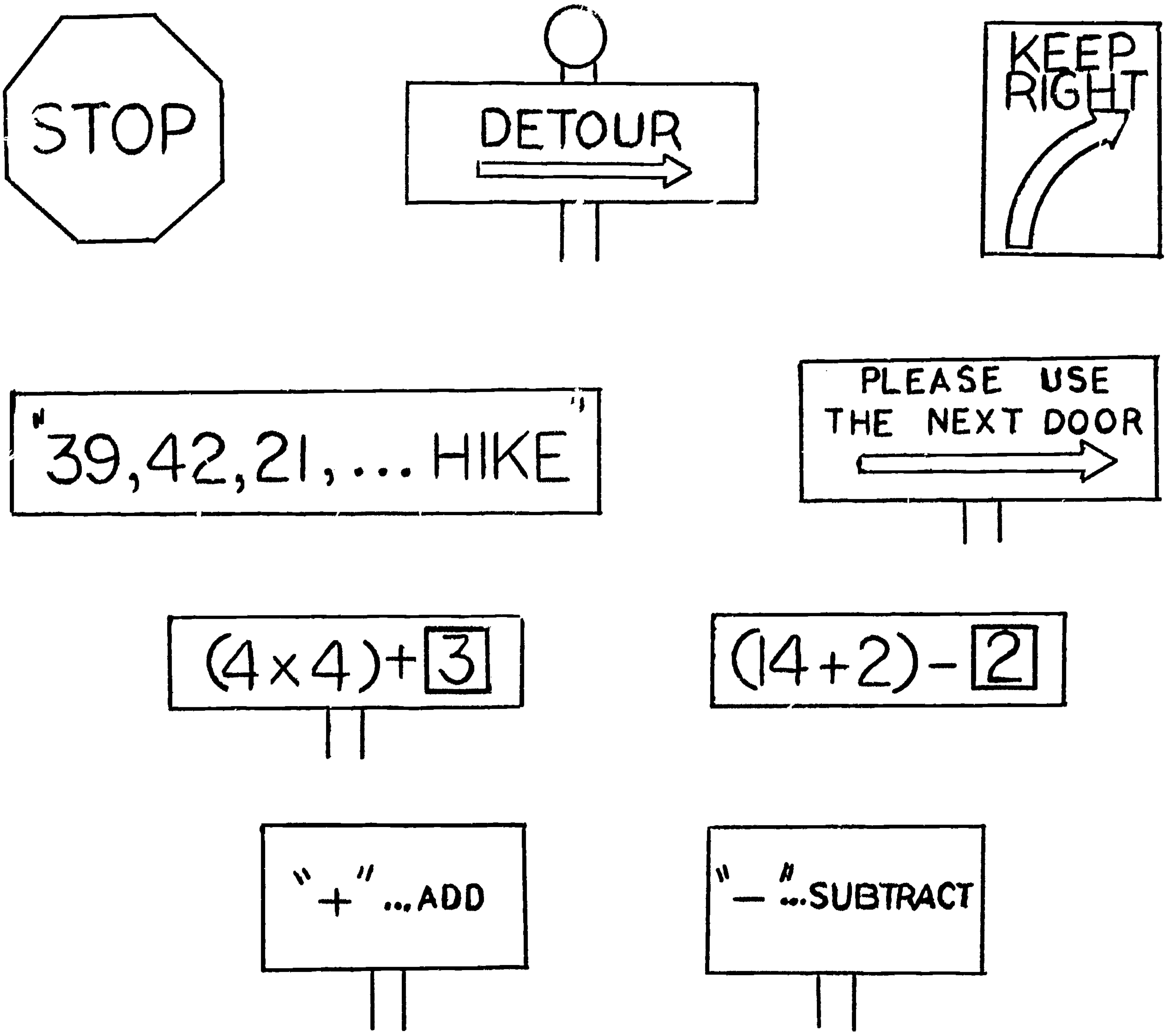
For information write: Dr. Jack L. Foley, Director
Bldg. S-503, School Annex
6th Street North
West Palm Beach, Florida

NUMBER SENTENCES

Table of Contents

NUMBER EXPRESSIONS

Cues for Actions



Each sign above provides a "cue" for an action or actions. As you read each sign, do you know "exactly" what actions to take? In mathematics it is important to know what actions are suggested and the "opposites" or inverses of certain actions. In the following activities see if you can give the "inverse" action of each action listed. The first is complete—as an example.

Activities

Match by drawing lines to the inverse actions.
One pair is matched as an example.

look up	face west
turn left	multiply by 3
go north	add 5
divide by 3	run downstairs
halve	look down
face east	turn right
run upstairs	go south
subtract 5	double

If the same action is taken on each number in column one to get a number in column two, can you guess the action?

Column One		Column Two
2	→	4
3	→	6
4	→	8
5	→	10

How could you take a number from Column Two and find its matching value in Column One? Going from Column One to Column Two we double (multiply by 2). The inverse action would be used going from Column Two to Column One.

Fill in the missing values and give the action and its inverse.

Double → (action)	← Halve (inverse)
4	8
6	$\square$
9	$\square$
$\square$	22
$\square$	16
n	$\square$
$\square$	x

Write in the action and its inverse, then fill in the missing values.

→ (action)	← (inverse)
3	6
4	7
$\square$	8
6	$\square$
$\square$	10
$\square$	14
m	$\square$
$\square$	x

2.

$\longrightarrow$ (action)	$\longleftarrow$ (inverse)
3	1
4	2
$\square$	6
6	$\square$
9	$\square$
$\square$	14
x	$\square$
$\square$	m

3.

$\longrightarrow$ (action)	$\longleftarrow$ (inverse)
x	$x + 4$
$x + 1$	$x + 5$
$x + 2$	$\square$
$\square$	$x + 8$
$x + 5$	$\square$
$\square$	$x + 14$

4.

$\longrightarrow$ (action)	$\longleftarrow$ (inverse)
2	8
4	16
$\square$	32
10	$\square$
$\square$	4
0	$\square$
x	$\square$
$\square$	n

Many actions have more than <u>one</u> step. Suppose an action has two steps and you are <u>to</u> "reverse" these steps. Sometimes these steps are called "doing" and reversing the steps is called "undoing." Would "doing" and "undoing" be inverse actions? If your answer is <u>yes</u>, then you are right.

Below are three examples of two or three step actions and their inverses. Look them over carefully.

Action		(reverse) Inverse Action	
<u>Step I</u>	Get shoes and socks	<u>Step I</u>	Take off shoes
<u>Step II</u>	Put on socks	<u>Step II</u>	Take off socks
<u>Step III</u>	Put on shoes	<u>Step III</u>	Put shoes and socks away

	Action		Inverse Action
Step I	Select some number n	Step I	From $3n + 2$ subtract $2 \longrightarrow 3n$
Step II	Multiply n by $3 \longrightarrow$ (result) $\longrightarrow 3n$	Step II	Divide $3n$ by $3 \longrightarrow n$
Step III	Add $2 \longrightarrow$ (result) $\longrightarrow 3n + 2$	Step III	You are back to the number selected

Use the steps above to fill in missing values.

n		$3n + 2$	
2	$\longrightarrow$	8	$(3 \times 2) + 2 = 8$
$\boxed{}$	$\longleftarrow$	11	$(11 - 2) \div 3 = 3$
4	$\longrightarrow$	$\boxed{}$	
$\boxed{}$	$\longleftarrow$	17	
8	$\longrightarrow$	$\boxed{}$	
$\boxed{}$	$\longleftarrow$	32	

An expression such as "$3n + 2$" is called an <u>open phrase</u> or open number expression. The value of the expression depends on the number value you assign n.

Suppose we have the expression $2n - 1$ (double n and subtract 1). For each value given for n, it is to be "doubled" and <u>one</u> subtracted. Fill in the missing values.

n	$2n - 1$
1	1
$\boxed{}$	7
3	$\boxed{}$
$\boxed{}$	15
9	$\boxed{}$

Activities

In each case below, give the steps used to get each expression. Reverse the steps and get back to the starting number.

Example: $2n + 4$

Steps to get the expression

1. Double n
2. Add 4

Reverse steps

$2n + 4 - 4 = 2n$ — 1. Subtract 4

$2n \div 2 = n$ — 2. Divide by 2 (halve)

1. $3n - 2$

Steps to get the expression

1.
2.

Reverse steps

$\longleftarrow$ 1.

$\longleftarrow$ 2.

2. $t + 75$

Step used

1.

Reverse step

$\longleftarrow$ 1.

As a class activity, state orally how to reverse the steps in each expression and leave only the letter (which represents some number) remaining.

1. $y - 2$
2. $t + 30$
3. $a + 12$
4. $3f$
5. $2p - 4$
6. $7m + 10$
7. $n \div 2$
8. $\frac{x}{3}$

9. $\frac{m}{3} + 6$

10. $15n - \frac{3}{4}$

11. $R + 3R$

12. $7m - 4m$

13. $2x + x$

14. $2x + \frac{1}{4}x$

Suppose we were filling in missing values as shown below. Notice only the values for $2n + 4$ are given.

n	2n + 4
	8
	20
	4
	6

We have to reverse steps to find values of n for each value for $2n + 4$. The reverse steps are:

1. Subtract 4
2. Divide by 2

For the first value of n we have:

$2n + 4 = 8$

$2n = 8 - 4$ <u>Step 1</u>

$n = (8 - 4) \div 2$ <u>Step 2</u>

$n = \boxed{2}$

This is true as: $(2 \times 2) + 4 = 8$

Find values for n that would make each statement true.

1. $2n + 4 = 20$
2. $2n + 4 = 4$

3. $2n + 4 = 6$ 4. $2n + 6 = 12$

Look back at the first problem. Suppose you covered up the 2n. You would have:

$$\boxed{} + 4 = 20$$

What number does 2n represent in the case above? It has to add to <u>4</u> and give <u>20</u>. What number plus 4 equals 20?

$$2n = 16$$

$$\text{or} \quad 2 \times n = 16$$

$$\text{cover n:} \quad 2 \times \boxed{} = 16$$

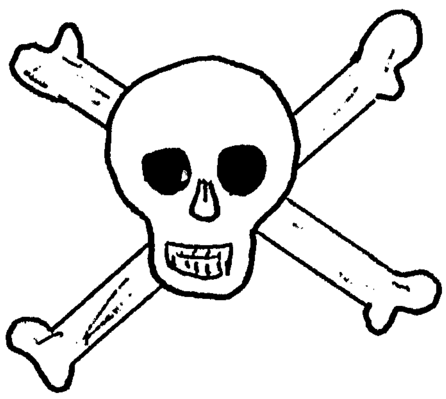
$$\text{Then } n = 8$$

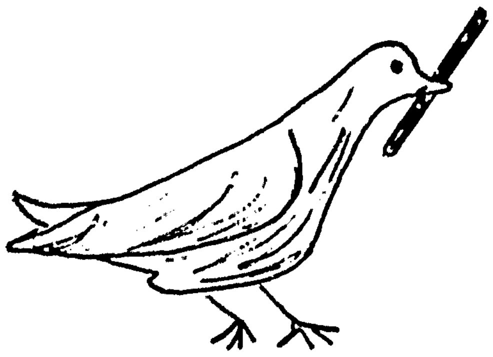
Try this with the other three problems above.

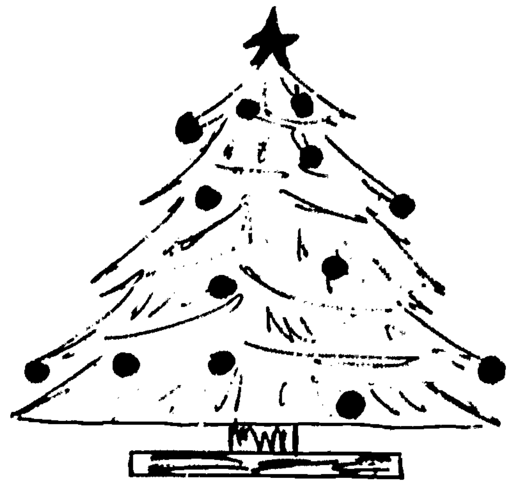
NUMBER SENTENCES

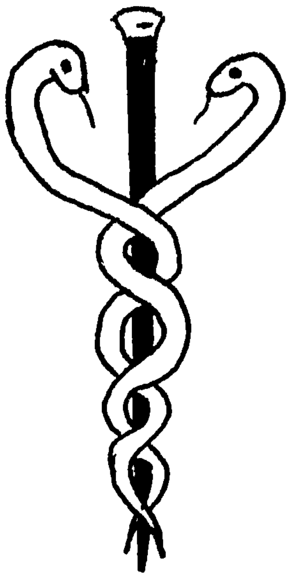
Symbols and Ideas

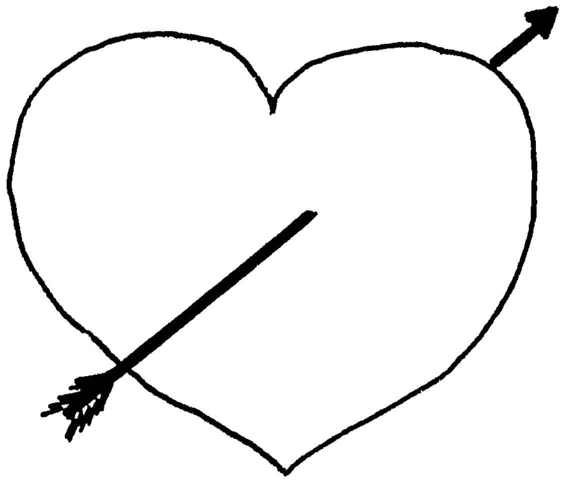
Have you ever thought about the way symbols or combinations of symbols are used to express ideas? What idea or thought is commonly expressed by these symbols?











These are pictorial (picture) symbols. However, a set of symbols can be made up and then used to express thoughts or ideas.

Consider the alphabet. This set of familiar symbols is:

{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w,x,y,z}

Suppose we select some of these symbols. Let's select the following:

{a, e, t}

These symbols can be "put together" in a <u>certain way</u> and "familiar words" are formed.

Examples: 1. eat

2. tea

These words can be "put together" in certain ways and <u>sentences</u> are formed. <u>Sentences</u> are used to express our thoughts.

How is all of this related to mathematics? As a matter of fact, sentences are very important in learning mathematics. Consider our basic set of symbols--called "numerals." <u>Numerals</u> are symbols used to express number ideas and the numerals are usually referred to as numbers. Actually "number" is the idea we try to relate and numerals are the symbols used.

This basic set is:

{0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

Suppose we select some of these symbols. Let's select

3, 5, 7

We put these together and form numbers.

Examples: 1. 357

2. 573

Now these numbers are used in writing number sentences. However, before we can write number sentences, we need "verb phrases" which in mathematics are called "relations." They are used in showing or stating how number or number expressions are related. Below are some important number relations. Both the symbol and its meaning are given:

Relations (verb phrases)

Symbol	Meaning
$=$	"is equal to"
$\neq$	"is not equal to"
$<$	"is less than"
$\not<$	"is not less than"
$>$	"is greater than"
$\not>$	"is not greater than"

The three basic symbols are: $=$, $<$, and $>$. A slash line drawn through means to put in the word "not." Did you notice that?

To better understand how "relations" are used in sentences, suppose some familiar relations are used.

Activities

In each sentence below underline the relation. The first is completed as an example.

1. John is taller than Bill.
2. Sue is a cousin of Novice.
3. Leroy is a friend of Wayne.
4. Six is greater than four.
5. Five is equal to three plus two.
6. Line AB is perpendicular to line CD.
7. People are funnier than monkeys.

8. Horses are smarter than mules.

9. Turtles are faster than rabbits.

10. Ten is equal to eight plus eight.

Would you say that all of the examples above are sentences? Are they all true? Can a sentence be either true or false? The answer is yes. Were you right?

Below are some false sentences--remember that they are sentences even though they are false.

1. You are ten feet tall.

2. Boys are smarter than girls.

3. Nine is less than four.

4. Six is equal to four minus 2.

5. All rectangles are squares.

Equations

Three basic "number relations" have been introduced.

They are:	$=$	"is equal to"
	$>$	"is greater than"
	$<$	"is less than"

A number sentence using the relation "is equal to" (=) is called an equation.

Equations can be true, false, or open (ask a question).

An example of each is given below:

A. $8 + 2 = 10$ True

B. $6 + 3 = 8$ False

C. $\square + 5 = 11$ Open

The last equation asks a question. What number added to five will give a sum of eleven? Open equations can be compared to regular sentences that ask questions such as:

1. "What" is the capital of Florida?

2. ____________ is the capital of the U. S.?

Each sentence above asks a question. Suppose the word "what" is replaced with some cities. For example:

A. "Pahokee" is the capital of Florida.

B. "Boca Raton" is the capital of Florida.

C. "Green Cove Springs" is the capital of Florida.

D. "Tallahassee" is the capital of Florida.

By replacing the word "what" with a city, a statement results that is either true or false--it is no longer open (a question). The object would be to supply a city that would make it a true statement. Are any of the statements above true? If so, the problem of naming the capital of Florida is solved.

In the illustration above, the word "what" could be replaced with cities and Tallahassee was the city that resulted in a true sentence.

Think of:

1. "What" as a variable.

2. Cities as members of a replacement set.

3. Tallahassee as the only member of the solution set. (A member of the replacement set that will make an open sentence a true statement.)

Act_vities

Write in the replacement set and the solution set for each of the open sentences. The first is an example.

1. "What" months begin with a J?

   Replacement set = {months of the year}

   Solution set = { January, June, July }

2. "What" day of the week begins with an S?

   Replacemant set = { }

   Solution set = { }

3. "Who" is the governor of Florida?

   Replacement set = {citizens of Florida}

   Solution set = { }

4. ____________________ is president of the U. S.

   Replacement set = { }

   Solution set = { }

5. "What" whole number plus five is equal to eight?

   Replacement set = {whole numbers}

   Solution set = { }

6. One-half of "what" whole number is equal to four?

   Replacement set = { }

   Solution set = { }

Open Equations

An open equation is a number sentence that asks a question. These sentences can be written using mathematical symbols or written verbally. An example is:

$$\square + 6 = 8$$

What number plus six "is equal to" eight?

Rather than use the frame ($\square$) as shown above, a letter of the alphabet is often used. Using some letter, say n:

$$n + 6 = 8$$

The letter, n, when used in an open number sentence is called a variable. It can be replaced by any selected number and the open sentence will become either a true or false statement. Suppose n is replaced by 4, then:

$$4 + 6 = 8$$

The resulting statement is false. If you can replace n with a number that will make the statement true, you have "solved" the equation. The number or numbers that will make the statement true belong to the solution set.

The replacement set is any number that can be used to replace the variable (regardless of whether it makes the sentence true or false).

Activities

For each open sentence below, give the number or numbers (solution set) that will make the sentence a true statement. Let s = solution set.

Replacement set = {only whole numbers}

1. $b + 6 = 24$; s = { }

2. $a + 3 = 10$; s = { }

3. $n - 3 = 15$; s = { }

4. $4 \times n = 20$; s = { }

5. $x \div 5 = 6$; s = { }

6. $18 - c = 6$; s = { }

7. $x^2 = 16$; s = { }

8. $13 - 9 = y$; s = { }

9. $2 \times y = 28$; s = { }

10. *$2n = 16$; s = { }

*2n "means" 2 x n. When a number is written next to a letter (variable) it means multiplication.

Many times number sentences are written out in words. No mathematical symbols are used. However, the verbal sentences can be rewritten using only mathematical symbols. See two examples below.

$$\underset{2}{\underline{\text{Two}}}\;\underset{\times}{\underline{\text{times}}}\;\underline{\text{some}}\;\underset{n}{\underline{\text{number}}}\;\underset{=}{\underline{\text{is}}}\;\underset{6}{\underline{\text{six}}}.$$

What is the solution? The answer is 3.

Activities

Rewrite each verbal number sentence making a number sentence using only mathematical symbols. Choose any letter to represent the unknown number. Give the solution for each sentence.

1. Some number plus eight is equal to nine. ______
2. What number times four equals twenty-four? ______
3. Twelve decreased by what number is seven? ______
4. Thirty-six divided by what number is three? ______
5. Sixteen increased by five is what sum? ______
6. Eighteen and how much more is twenty-eight? ______
7. One-half of what number is equal to sixteen? ______
8. Six multiplied by what number is equal to zero? ______
9. Five times what number is equal to ten plus five? ______
10. What number can you triple, then add three to it and get a result of fifteen? ______

Combining "like" terms

Suppose we have two or more numbers to be added or subtracted and each multiplied by the same variable.

For example: $3n + 2n$

By adding the "numbers" only--not the letters:

$$3n + 2n = 5n$$

Replace n with some different values and check the equation above.

$n = 6$: $3n + 2n = 5n$ or $5 \times 6 = 30$

(another method) $(3 \times 6) + (2 \times 6) =$

$18 + 12 = 30$

$n = 8$: $3n + 2n = 5n$ or $5 \times 8 = 40$

(another method) $(3 \times 8) + 2 \times 8) =$

$24 + 16 = 40$

Activities

Combine "like" terms.

1. $5n + 6n = \boxed{}$

2. $9y - 2y = \boxed{}$

3. $2n + 4n - 3n = \boxed{}$

4. $4n + \boxed{} = 11n$

5. $6p + 4p - \boxed{} = 2p$

6. $16r + 2r - 11r = \boxed{}$

Use the idea of "combining" like terms and solve each equation. The first is an example. Check each answer.

7. (example)

$$\begin{aligned} 3n + 2n &= 18 - 3 \\ 5n &= 15 \\ n &= 3; \quad s = \{3\} \end{aligned}$$

check:

$$\begin{aligned} (3 \times 3) + (2 \times 3) &= 18 - 3 \\ 9 \quad + \quad 6 &= 15 \\ \underline{15} &= \underline{15} \end{aligned}$$

8. $6n - 2n = 24$ check:

9. $9n + 2n - 8n = 21 - 3$ check:

10. $12y - 6y = 36$ check:

Writing Equivalent Equations

Below are three equations. Which would you rather solve?

1. $3n + 2n + 4 = 29$
2. $3n + 2n = 25$
3. $5n = 25$

The last equation is very easy to solve. The sentence asks, "Five times what number is equal to twenty-five?" The answer (solution) is five. The solution for the first two equations is also five. These three equations have exactly the same solution.

Equivalent Equations are equations with exactly the same solution.

It is important to know how to write equivalent equations. If an equation is "inspected" and it is a little too complicated (difficult) to find the answer by inspection, an "equivalent equation" that is easier to solve can be written. This can be continued until you get an equation that can be solved by inspection. Since each of the equations has the same solution, if you solve the "easy" equation, then you also have the answer to the "difficult" equation.

Now the question is: "What ideas can be used to write equivalent equations?"

IDEA I: Add or subtract the same value from each side of the equation.

IDEA II: Multiply or divide each term of the equation by the same value (not zero).

The first idea is used to get all terms that have a variable on the "left" side of the equation and all number values (not a variable term) on the "right" side.

Examples

A.

(Subtract 3 from each side)

$$a + 3 = 10$$
$$a + 3 - 3 = 10 - 3$$
$$a + 0 = 7$$
$$a = 7$$

check:

$7 + 3 = 10$ True

B.

(Add 6 to each side)

$$\begin{aligned} y - 6 &= 9 \\ y - 6 + 6 &= 9 + 6 \\ y - 0 &= 15 \\ y &= 15 \end{aligned}$$

Check:

$15 - 6 = 9$ True

C.

(Add 4)

(Subtract y)

$$\begin{aligned} 2y - 4 &= 9 + y \\ 2y - 4 + 4 &= 9 + y + 4 \\ 2y - 0 &= 13 + y \\ 2y - y &= 13 + y - y \\ y &= 13 \end{aligned}$$

This could have been done in one step. As:

(Add 4 and subtract y)

$$\begin{aligned} 2y - 4 &= 9 + y \\ 2y - y - 4 + 4 &= 9 + y - y + 4 \\ y &= 13 \end{aligned}$$

Check:

$$\begin{aligned} (2 \times 13) - 4 &= 9 + 13 \\ 26 - 4 &= 22 \\ 22 &= 22 \end{aligned}$$

True

Activities

Solve each equation by the addition or subtraction idea. Check each. Combine like terms if necessary.

1. $b + 6 = 24$ Check:

2. $n - 3 = 15$ Check:

3. $c + 12 = 15$ Check:

4. $w - 14 = 21$ **Check:**

5. $3y + 2 = 6 + 2y$ **Check:**

6. $6y - 5y + 4 = 10$ **Check:**

In using Idea II, notice that each term of an equation must be multiplied or divided by the same number (not zero). This idea is used to "isolate" or leave only the variable.

For example, if we have an expression, say:

$$5n$$

then $n = 5n \div 5$

or an expression such as:

$\frac{n}{5}$ $(n \div 5)$

then $n = \frac{n}{5} \times 5$

Consider the forms of the equations in the examples below.

Example I. $5 \times \boxed{} = 30$

This can be written as: $5n = 30$

(Divide each term by 5) $\frac{5n}{5} = \frac{30}{5}$

$n = 6$ **Check:** $5 \times 6 = 30$

Example II. $\frac{n}{2} = 30$

Multiply each term by 2 $\frac{2n}{2} = 30 \times 2$

$n = 60$ **Check:** $\frac{60}{2} = 30$

Activities

Multiply or divide each term by some value and solve for the variable. Check each answer.

1. $2n = 18$

2. $\frac{n}{3} = 15$ (Can be written as: $\frac{1}{3}n = 15$ or $n \div 3 = 15$)

3. $6r = 42$

4. $\frac{1}{5}q = 10$

In the equations below:

First, combine any "like" terms if necessary.

Second, add or subtract to get the variable on the "left" side and the "right" side--again combine any "like" terms.

Third, multiply or divide each term left in the equation and solve for the variable.

Example: $2y + 3y - 2 = y + 18$

Combine like terms. $5y - 2 = y + 18$

Add 2 and subtract y. $5y - y - 2 + 2 = y - y + 18 + 2$

$4y = 20$

Divide by 4. $\frac{4y}{4} = \frac{20}{4}$

$y = \underline{\underline{5}}$

Check:

$(2 \times 5) + (3 \times 5) - 2 = 5 + 18$

$10 + 15 - 2 = 23$

$23 = 23$

5. $8n + 2n = 120$

6. $25r - 15r = 200$

7. $6p + 6 = 368 + 2p$

8. $3y + 2y + y = 12 + 6$

Supplementary Exercises

Use the ideas you have learned to solve each of the following equations. Discuss and solve these as a class activity. State orally how to solve each. State each as a verbal question.

1. $4a = 16$	2. $3c = 36$
3. $y + 2 = 10$	4. $24 + 3n = 216$
5. $8n + 22 = 126$	6. $\frac{n}{3} = 3$
7. $\frac{a}{13} = 2$	8. $25 = \frac{d}{3}$
9. $\frac{p}{10} = 100$	10. $w - 17 + 2w = 212$
11. $3y + y + 26 = 166$	12. $2n = 5$
13. $\frac{1}{3}n + \frac{2}{3}n = 9$	14. $5q + 21 = 266$
15. $\frac{n}{4} = \frac{1}{2}$	16. $\frac{q}{9} = \frac{2}{3}$

INEQUALITIES

Number sentences using the relation (=) "is equal to" were called an equation. Number sentences using the relation (>) "is greater than" or (<) "is less than" are called inequalities.

An inequality can be true, false, or open. An example of each is given below.

1.	$7 > 5$	Seven is greater than five.	True
2.	$16 < 7$	Sixteen is less than seven.	False
3.	$\boxed{} > 2$	Some number is greater than two.	Open

Did you notice that when the number statement is true for an inequality relation, the point of the symbol is toward the smaller of the two numbers?

Activities

Exercise 1. In the following exercise, tell whether each relation is true or false. Read each as a verbal sentence.

		True	False
1.	$10 > 5$	______	______
2.	$1 < 2$	______	______
3.	$5 < 10$	______	______
4.	$6 > 6$	______	______
5.	$2 + 1 > 2 + 2$	______	______
6.	$27 < 72$	______	______
7.	$345 < 354$	______	______
8.	$10 - 2 < 4 + 5$	______	______
9.	$3 + 14 > 12 - 4$	______	______
10.	$\frac{3}{4} + \frac{5}{4} > \frac{5}{8} + \frac{11}{8}$	______	______
11.	$13 + 3 < 15 + 2$	______	______
12.	$1 + 2 + 3 > 4 + 1$	______	______

Exercise 2. Write the symbol > or < to make each number relation true.

Examples

a) $3 > 2$ Three "is greater than" two, therefore the symbol > is used to make the relation true.

b) $2 < 3$ Two "is less than" three, therefore the symbol < is used to make the relation true.

1. $31 \; \square \; 13$
2. $7 \; \square \; 8$
3. $\frac{1}{2} \; \square \; \frac{1}{4}$
4. $8 \times 8 \; \square \; 8 \times 9$
5. $14 - 2 \; \square \; 10 + 3$
6. $22 - 8 \; \square \; 2 \times 8$
7. $32 + 8 \; \square \; 51 \div 17$
8. $0.4 \; \square \; 0.04$
9. $56 \; \square \; 62$
10. $239 \; \square \; 2.39$
11. $56 \; \square \; 6.3$
12. $190 \; \square \; 109$
13. $84.7 \; \square \; 847$
14. $5 \times 4 \; \square \; 25 \div 5 + 3$

Note: On the number line, the smaller of two numbers is to the left of the larger number.

Variables are used in an inequality relation just as they are used in an equation relation. The solution set of the variable is the values that would make the sentence true. Consider the inequality below.

Example a) $x + 2 > 5$

How many values for x can be used? Solve the inequality as you would an equation. Subtract 2 from both sides of the inequality.

$$x + 2 - 2 > 5 - 2$$
$$x + 0 > 3$$
$$x > 3$$

If x "is greater than" 3, then the inequality $x + 2 > 5$ is true. Any number to the right of 3 on the number line "is greater than" three. The solution set then is all numbers greater than three.

Example b. $t - 7 > 3$

Solve the inequality in a like manner as you would an equation by adding seven to both sides of the inequality.

$$t - 7 + 7 > 3 + 7$$

$$t + 0 > 10$$

$$t > 10$$

If t "is greater than" 4 then the inequality $7 > 3$ is true. Any number to the right of 4 on the number line "is greater than" 4. The solution set is <u>all</u> numbers greater than 4.

Exercise 3. Inequalities in addition. Don't forget cues to actions.

1. $t + 7 < 15$
2. $a + 15 > 45$
3. $3y + 3 > 27$
4. $\frac{2}{3}h + 9 > 21$
5. $2y + 15 > 105$
6. $7w + 3 > 2w + 18$
7. $12t + 21 < 8t + 69$
8. $24 > 3t$
9. $96p + 105 > 681$
10. $36 > 6t$

Using the set of whole numbers, $\{0, 1, 2, 3...\}$ list a set of elements that will satisfy each inequality and write a statement to represent the same set.

Example

$$b + 7 < 15$$

$$b + 7 - 7 < 15 - 7$$

$$b < 8$$

Set notation from the whole number

$\{7, 6, 5, 4, 3...\}$

Statement

"Seven and all whole numbers smaller than seven"

All of the inequalities from the above exercise used the inverse operation of addition, which is the operation of subtraction. If any of these problems gave "trouble," check them for inventory review.

Exercise 4. Inequalities with subtraction. Solve each inequality and write a statement to express the replacement set.

1. $b - 7 < 15$

2. $a - 15 < 45$

3. $a - 15 > 45$

4. $3y - 3 > 27$

5. $\frac{2}{3}h - 9 > 21$

6. $18x - 13 < 6x + 11$

7. $2.5p + 3.2p > 36.8 - 14$

8. $\frac{7}{8}b - \frac{5}{8}b > \frac{3}{4}$

9. $\frac{9}{10} < \frac{1}{2}c - \frac{1}{4}$

10. $7.5w - 2.3 < 5.2$

11. $38 - 3 > 7t$

12. $K - 27 < 27$

13. $C - 27 > 27$

Review problems 2, 3, 12 and 13. Do you notice anything about the replacement set?

NUMBER SENTENCES

Illustration of Terms

Combine Like Terms

Original Expression	Expression after combining like terms.
a) $3n + 4n$	$7n$
b) $2n + 3n - 6$	$5n - 6$
c) $2n - 8p + 2p + 10n + 3$	$12n - 6p + 3$
d) $4 + 2n + 1 - 7n + 6n$	$n + 5$

Decreased--made smaller; if a number is decreased by another number it means the second number is subtracted from the first.

Word sentence: Eight decreased by six is equal to two.

Number sentence: $8 - 6 = 2$

Word sentence: Forty decreased by ten is equal to thirty.

Number sentence: $40 - 10 = 30$

Equation--a number sentence using the relation "is equal to".

Examples:

$6 \div 3 = 2$ $\quad$ $7 \times 3 = 21$

$4 + 2 + 6 = 12$ $\quad$ $7 + 6 - 10 = 3$

Equivalent Equations-- equations with exactly the same solution.

Equations a), b), and c) are equivalent equations

because $n = 3$ is a solution to each of them.

a) $20n = 60$ $\quad$ b) $20n - 10 = 50$ $\quad$ c) $12n + 8n = 60$

Equations d), e), f), and g) are equivalent equations because $p = 2$ is a solution to each of them.

d) $18p + 6 + 36p = 114$ $\quad$ f) $3p + 6p = 18$

e) $18p + 36p = 108$ $\quad$ g) $9p = 18$

Equations h), i), and j) are equivalent equations because $x = 3$ is a solution to each of them.

h) $25x + 75x - 50 = 250$ i) $25x + 75x = 300$ j) $100x = 300$

Increased--made larger; if a number is increased by another number it means the second number is added to the first.

Word sentence: Five increased by four is equal to nine.

Number sentence: $5 + 4 = 9$

Inequality--number sentences using the relations (>) "is greater than" or (<) "is less than" are called inequalities.

Inverse action--the undoing or opposite of an action; subtraction is the inverse of addition. Going up is the inverse of going down.

Numerals--symbols used to express number ideas. Numerals are usually referred to as numbers.

Number--the idea we try to relate by the use of the symbols called numerals.

Number sentences--a sentence which is made up of numerals and a relation.

$6 + 4 = 10$; $35 + 2 > 29$; $17 - 6 < 40$

Open Equation--an open equation is a number sentence that asks a question. These sentences can be written using mathematical symbols or written verbally. Notice the examples below.

1. $\square - 7 = 15$

2. What number minus seven "is equal to" fifteen?

Open Number Expression--a group of mathematical symbols which does not contain a relation.

$6 + n$; $8n + 4$; $3n - 10$

Open Phrase--see open number expression

<u>Relations</u>--expressions which tell us if one number or group of numbers is bigger than, smaller than, or the same size as another number or group of numbers.

symbol	meaning
$=$	"is equal to"
$\not<$	"is <u>not</u> less than"

<u>Replacement Set</u>--the set of elements from which we may choose the answer to a question.

Question: What is the largest state in the United States?

Replacement Set--the set of the fifty states.

<u>Solution Set</u>--the set of elements which are correct answers to a question.

Question: Which even numbers are less than ten?

Replacement Set--all even numbers.

Solution Set--2, 4, 6, 8.

<u>Term</u>--any number or any variable within an equation is called a term.

$6n + 40 = 100$	6n, 40 and 100 are terms
$75n - 5 = 70n$	75n, 5 and 70n are terms

<u>Variable</u>--the unknown quantities in a number sentence.

$n + 14 = 20$	n is the variable term; 14 and 20 are constant terms.
$4n + 7 = 40$	n is the variable term; 4, 7, and 40 are constant terms.
$12n + 8n = 60$	n is a variable. What do you think the constant terms are?